NAG Toolbox for MATLAB

g01fa

1 Purpose

g01fa returns the deviate associated with the given probability of the standard Normal distribution, via the function name.

2 Syntax

```
[result, ifail] = g01fa(tail, p)
```

3 Description

The deviate, x_p associated with the lower tail probability, p, for the standard Normal distribution is defined as the solution to

$$P\left(X \leq x_p\right) = p = \int_{-\infty}^{x_p} Z(X)\, dX,$$

where

$$Z(X) = \frac{1}{\sqrt{2\pi}} e^{-X^2/2}, \qquad -\infty < X < \infty.$$

The method used is an extension of that of Beasley and Springer 1977. p is first replaced by $q = p - 0.5$.

(a) If $|q| \leq 0.3$, x_p is computed by a rational Chebyshev approximation

$$x_p = s\frac{A\left(s^2\right)}{B\left(s^2\right)},$$

where $s = \sqrt{2\pi}q$ and A, B are polynomials of degree 7.

(b) If $0.3 < |q| \leq 0.42$, x_p is computed by a rational Chebyshev approximation

$$x_p = \operatorname{sign} q\left(\frac{C(t)}{D(t)}\right),$$

where $t = |q| - 0.3$ and C, D are polynomials of degree 5.

(c) If $|q| > 0.42$, x_p is computed as

$$x_p = \operatorname{sign} q\left[\left(\frac{E(u)}{F(u)}\right) + u\right],$$

where $u = \sqrt{-2 \times \log(\min(p, 1-p))}$ and E, F are polynomials of degree 6.

For the upper tail probability $-x_p$ is returned, while for the two tail probabilities the value x_{p^*} is returned, where p^* is the required tail probability computed from the input value of p.

4 References

Abramowitz M and Stegun I A 1972 *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Beasley J D and Springer S G 1977 Algorithm AS111. The percentage points of the normal distribution *Appl. Statist.* **26** 118–120

Hastings N A J and Peacock J B 1975 *Statistical Distributions* Butterworths

5 Parameters

5.1 Compulsory Input Parameters

1: **tail – string**

Indicates which tail the supplied probability represents.

tail = 'L'

The lower probability, i.e., $P(X \leq x_p)$.

tail = 'U'

The upper probability, i.e., $P(X \geq x_p)$.

tail = 'S'

The two tail (significance level) probability, i.e., $P(X \geq |x_p|) + P(X \leq -|x_p|)$.

tail = 'C'

The two tail (confidence interval) probability, i.e., $P(X \leq |x_p|) - P(X \leq -|x_p|)$.

Constraint: **tail** = 'L', 'U', 'S' or 'C'.

2: **p – double scalar**

p, the probability from the standard Normal distribution as defined by **tail**.

Constraint: $0.0 < \mathbf{p} < 1.0$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

If on exit **ifail** $\neq 0$, then g01fa returns 0.0.

ifail = 1

On entry, **tail** $\neq$ 'L', 'U', 'S' or 'C'.

ifail = 2

On entry, $\mathbf{p} \leq 0.0$,
or $\mathbf{p} \geq 1.0$.

7 Accuracy

The accuracy is mainly limited by the ***machine precision***.

8 Further Comments

None.

9 Example

```
tail = 'L';
p = 0.975;
[result, ifail] = g01fa(tail, p)
```

```
result =
    1.9600
ifail =
           0
```